



Open Archive TOULOUSE Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <http://oatao.univ-toulouse.fr/>
Eprints ID: 19944

To link to this article: DOI: 10.1109/20.376290
URL: <http://dx.doi.org/10.1109/20.376290>

To cite this version : Charpentier, Jean-Frédéric and Lefèvre, Yvan and Sarraute, Emmanuel and Trannoy, Bernard *Synthesis and modelling of an electrostatic induction motor*. (1995) IEEE Transactions on Magnetism, vol. 31 (n° 3). pp. 1404-1407. ISSN 0018-9464

Any correspondence concerning this service should be sent to the repository administrator: staff-oatao@listes-diff.inp-toulouse.fr

SYNTHESIS AND MODELLING OF AN ELECTROSTATIC INDUCTION MOTOR

J.F. Charpentier, Y. Lefèvre, E. Sarraute and B. Trannoy

LEI/ENSEEHT

2, Rue Charles Camichel
31071 Toulouse Cédex FRANCE

Abstract -- This paper deals with a new way of synthesis and modelling electrostatic induction micromotors by means of duality rules from the magnetic induction machine. An electromechanical model based on this method is given. Then, a computational procedure based on a general lumped parameter model and an electric field calculation code, has been developed so as to simulate the dynamic working of these actuators. A comparison is made between the computation results and the model results. Satisfactory agreement between theory and simulation is obtained in most respects.

I. INTRODUCTION

Research on a new type of actuators have been undertaken for a few years: the electrostatic microactuators. In these actuators the electromechanical conversion is based on the electric field rather than the magnetic field. Like the classical magnetic machines, electrostatic machines can work on different principles: electrostatic synchronous motor (E.S.M.), electroquasistatic induction motor (E.I.M.), variable capacitance motor (V.C.M.). For instance the V.C.M. have been widely studied by many researchers: in the U.S.A., Japan and Europe. Some prototypes have been designed and fabricated using integrated-circuit processing.[1][2][3]

This paper is concerned with the E.I.M. Some authors have already studied this type of machine and developed theoretical models based on the theory of electromagnetic waves. These models are mainly adapted for E.I.M. with the rotor made of smooth uniform conductor or a fluid and are not very easy to manipulate for the design. This type of E.I.M. uses charge relaxation to establish its rotor charge distribution. It is therefore difficult to optimise its performances.[4][5]

The paper shows how to synthesize another type of E.I.M. from considerations of duality with the familiar magnetic induction machine (M.I.M.). In order to facilitate the optimisation of this kind of motor Park's equations for this synthesised E.I.M. are inferred from some hypotheses. Eventually a general lumped parameter model is given. This last model is used to simulate dynamic working of E.I.M. and to validate the obtained Park's model.

II. SYNTHESIS OF THE MACHINE

In theory the E.I.M. is a close analog of its familiar electromagnetic counterparts. Voltages and electric field play the parts of currents and magnetic field. The qE term in Lorentz force takes place of the $j \times B$ Laplace force. The conception of our model is based on this basic idea and considerations of duality with the familiar M.I.M. and on

some previous works on the conception of micrometric actuators [6][7][8].

An E.I.M. requires a revolving electric field on its stator. Such as in the M.I.M. this is achieved by applying a three-phased voltages source to electrodes equally spaced around the stator. In order to create an alternating electric field distribution each phase consists of pair of electrodes feeded with opposite sinusoidal voltages (Fig. 1). Figure 2 shows the final arrangement of the stator of a triphased, two poles machine.

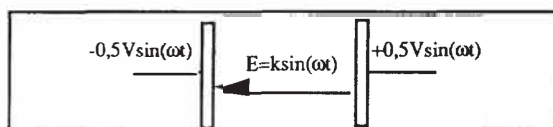


Fig 1. Creating an alternating electric field

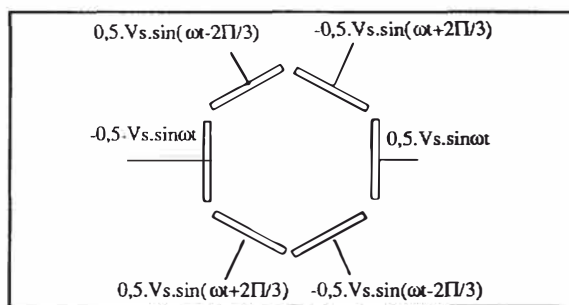


Fig. 2. Creating a rotating electric field

In a M.I.M., due to Lenz's law combined with Ohm's law, the revolving magnetic field induces voltages and currents in the rotor windings. According to Lenz's law these currents produce magnetic effects which counterbalance the evolution of flux in the rotor coil. In a wound rotor machine the rotor coils are short-circuited. Relations between the flux Φ_r and the current I_r in a coil are:

$$e = -d\Phi/dt \text{ and } I_r = e/R \quad (1)$$

where e is the e.m.f.

An equivalent induction phenomena can be achieved by means of electric induction. In an E.I.M. the revolving electric field induces voltages and charges on the rotor electrodes. By considerations of duality from the wound rotor M.I.M., the phases of the rotor of an E.I.M. can consist of isolated pairs of electrodes linked with a resistor (Fig. 3).

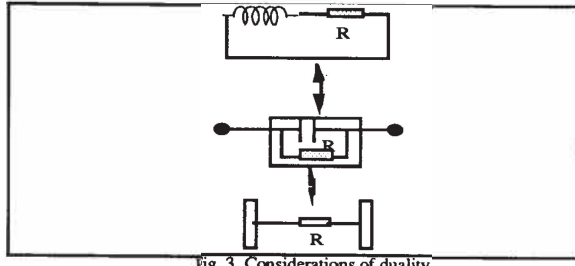


Fig. 3. Considerations of duality

Relations between the charge Q_r and the voltage V_r between the two electrodes forming a phase are:

$$I_r = dQ_r/dt \text{ and } V_r = -R_r / I_r \quad (2)$$

These relations show that the induced voltages counterbalance the evolution of charge on electrodes. The current I_r between the two electrodes play the part of the e.m.f. in M.I.M. and can be called current motive force.

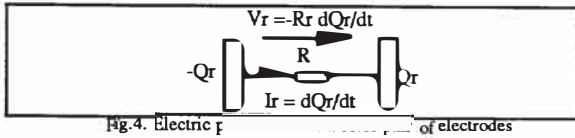


Fig. 4. Electric field of electrodes

To illustrate our approach we have chosen a three-phased rotor E.I.M. One arrangement of a two poles motor is shown on figure 5.

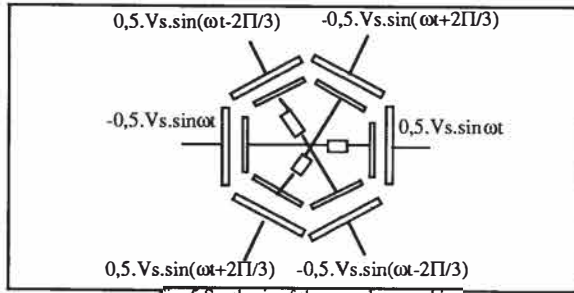


Fig. 5 Synthesis of the complete machine

III. PARK'S EQUATIONS FOR THE SYNTHESIZED E.I.M

In order to deduce a relatively simple model of this E.I.M. few assumptions must be made: space harmonics and saliency effects are neglected. Thus for three-phased E.I.M. the matrix capacitance is:

$$\begin{bmatrix} Q_{s1} \\ Q_{s2} \\ Q_{s3} \\ Q_{r1} \\ Q_{r2} \\ Q_{r3} \end{bmatrix} = \begin{bmatrix} C_s & \frac{M_s}{2} & \frac{M_s}{2} & a & b & c \\ \frac{M_s}{2} & C_s & \frac{M_s}{2} & c & a & b \\ \frac{M_s}{2} & \frac{M_s}{2} & C_s & b & c & a \\ a & c & b & C_r & \frac{M_r}{2} & \frac{M_r}{2} \\ b & a & c & \frac{M_r}{2} & C_r & \frac{M_r}{2} \\ c & b & a & \frac{M_r}{2} & \frac{M_r}{2} & C_r \end{bmatrix} \begin{bmatrix} V_{s1} \\ V_{s2} \\ V_{s3} \\ V_{r1} \\ V_{r2} \\ V_{r3} \end{bmatrix} \quad (3)$$

where Q_{si} , Q_{ri} , V_{si} , V_{ri} are the charge and the voltages of the phase on the stator (s) and on the rotor (r); C_s and C_r are the coefficients capacitance of a phase on the stator and on the rotor; M_s is the coefficient of induction between two phases on the stator; M_r is the coefficient of induction between two phases on the rotor; a, b and c are given by:

$$\begin{aligned} a &= -M_{sr} \cos(\theta) \\ b &= -M_{sr} \cos(\theta + 4\pi/3) \\ c &= -M_{sr} \cos(\theta + 2\pi/3) \end{aligned} \quad (4)$$

where M_{sr} is the amplitude of the coefficients of induction between a phase on the stator and a phase on the rotor, and θ the electrical angular position of the rotor located from the phase numbered one of the stator.

As for the M.I.M. the coefficients of equation (3), relating charges to voltages, are function of the angular position of the rotor θ . In order to simplify these relations the three-phased variables are transformed to new variables related to a reference frame fixed to the revolving field. This reference frame is the analog of the d-q axis and the transformation is the classical Park's transformation. Equation (3) becomes:

$$\begin{cases} Q_s = C_{ss} V_s - m_{sr} V_r \\ Q_r = C_{rr} V_r - m_{sr} V_s \end{cases} \quad (5)$$

where:

$$C_{ss} = C_s - M_s/2, \quad C_{rr} = C_r - M_r/2 \text{ and } m_{sr} = 3M_{sr}/2 \quad (6)$$

Equation (5) relates the complex values of the new electrical variables to one another. Under steady conditions and assuming balanced three-phase sinusoidal feeding voltages applied on the phases of the stator, the electrical pulsation ω_s of the stator variables and the electrical pulsation ω_r of the rotor variables are related to the electrical angular speed of the rotor by the equation:

$$\omega_r + \omega = \omega_s \quad (7)$$

This equation is the non-zero torque condition. In these conditions all the new variables are constant in the new reference. The current of a phase of the rotor in the new reference is:

$$I_r = \frac{dQ_r}{dt} - j \frac{d\alpha}{dt} Q_r = -j \frac{d\alpha}{dt} Q_r = j\omega_r (C_{rr} V_r - m_{sr} V_s) \quad (8)$$

From equations (2) and (5) the expression of voltage of a rotor phase is deduced:

$$V_r = \frac{j\omega_r m_{sr} V_s}{[j\omega_r C_{rr} + 1/R_r]} \quad (9)$$

We can compare the equation (9) with the classic expression for the magnetic induction machine.

$$I_r = \frac{j\omega_r m_{sr} I_s}{[j\omega_r L_{rr} + R_r]} \quad (10)$$

The torque is obtained by the derivative of the electric coenergy in the machine with respect to rotor position. It's expression is:

$$\Gamma = \frac{1}{2} [V]^T \frac{d[C(\theta)]}{d\theta} [V] \quad (11)$$

where $[V]$ is the vector of voltages of the phases of the motor $[C(\theta)]$ is the matrix of capacitances represented in equation (3). This matrix can be written by 4 blocs:

$$[C(\theta)] = \begin{bmatrix} [C_{ss}] & [M_{sr}(\theta)] \\ [M_{sr}(\theta)]^T & [C_{rr}] \end{bmatrix} \quad (12)$$

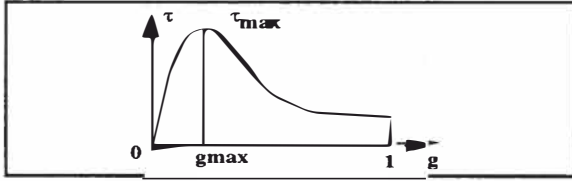
Neglecting saliency effects, C_{ss} and C_{rr} are not functions of rotor position. From equations (11) and (12) the expression of torque is:

$$\Gamma = \frac{1}{2} [V_s]^T \frac{d[M_{sr}(\theta)]}{d\theta} [V_r] = -2 \cdot m_{sr} \cdot \text{Im}(V_r V_s^*) \quad (13)$$

Assuming steady states operations and balanced sinusoidal voltages supply, V_s can be supposed real and equation (13) becomes:

$$\Gamma = \frac{2 \cdot m_{sr}^2 \cdot V_s^2 \cdot \omega_r / R_r}{1 / R_r^2 + \omega^2 \cdot C_{rr}^2} = \frac{2 \cdot m_{sr}^2 \cdot V_s^2 \cdot \omega_s / R_r}{1 / (R_r^2 \cdot g) + g \cdot \omega_s^2 C_{rr}^2} \quad (14)$$

Where $g = \omega_r / \omega_s$ is the slip of the machine. The torque-slip characteristic is qualitatively similar to M.I.M. one's (Fig. 6).



IV. GENERAL LUMPED-PARAMETER ELECTROMECHANICAL DYNAMIC

To infer Park's model of the E.I.M., assumptions are needed. In order to validate this model a more general model is elaborated. The general differential system of equations describing the dynamic working of the synthesised E.I.M. can be written in the form:

$$\begin{cases} \frac{d[V_r]}{dt} = [C_{rr}]^{-1} \cdot \left[-[R_r]^{-1} [V_r] - \frac{d[C_{rr}]}{d\theta} \omega [V_r] \right. \\ \left. - [M_{sr}] \frac{d[V_s]}{dt} - \frac{d[M_{sr}]}{d\theta} \omega [V_s] \right] \\ J \frac{d\omega}{dt} = \Gamma_e - t_{fs} \cdot \text{signe}(\omega) - t_{fv} \cdot \omega - \Gamma_m \\ \frac{d\theta}{dt} = \omega \end{cases} \quad (15)$$

where Γ_e is the electrical torque given by (11); t_{fs} , t_{fv} , J and Γ_m are respectively the viscous and static friction torque coefficients, the inertia and the load torque; $[V_s]$ and $[V_r]$ are the vector of the voltages on the stator and on the rotor. $[C_{rr}]$, $[M_{sr}]$ are the matrix blocs components of the matrix of coefficients of capacitance and induction $[C(\theta)]$. The components of $[C(\theta)]$ are function of the rotor position.

The $[C(\theta)]$ matrix is determined by using an electric field computation code based on finite elements method. This code can take into account the movement of the rotor by means of the moving band technique [7][9]. Using this code and considering symmetry, all the components of the matrix $[C(\theta)]$ can be determined by only two simulations.

The differential system (15) is solved by means of a Runge-Kutta method. For any given voltages supply waveforms, all the electromechanical variables are determined as a function of time.

With this procedure the working of a triphased, 6 poles E.I.M. has been simulated. The arrangement of the structure of the machine is presented on figure 7. The diameter of the rotor is 120 μm .

First the components of the matrix $[C(\theta)]$ have been calculated. When only a unit volt is applied on a reference phase of the machine and zero on the other phases, the calculation of charges on each phase gives the coefficient of capacitance on the reference phase and the coefficients of induction between the reference phase and the others phases. Figure 7 shows the map of the electric field calculated inside the machine when the reference phase is a stator one's. The evolution of some coefficients versus the rotor position are shown on figure 8. The dotted line represents the fundamental harmonic of one of the coefficients of induction between stator and rotor. From these results, the parameters of the Park's model are deduced:

$$\begin{aligned} C_{ss} &= 0.1226 \cdot 10^{-15} \text{ F} \\ C_{rr} &= 0.2238 \cdot 10^{-15} \text{ F} \\ m_{sr} &= 0.885 \cdot 10^{-16} \text{ F} \end{aligned} \quad (16)$$

During the dynamic simulation the machine is fed by a three-phased balanced sinusoidal voltages system with an amplitude of 200 volts and a frequency of 1 kHz. The values of the others parameters are:

$$\begin{aligned} t_{fs} &= 0; & t_{fv} &= 8 \cdot 10^{-16} \\ \Gamma_m &= 8 \cdot 10^{-13} \text{ N.m} & J &= 40 \cdot 10^{-20} \text{ kg m}^2 \end{aligned} \quad (17)$$

We have simulated the starting of the machine from standstill to a 1900 rad/s steady state. The results are presented on figure 9. The voltages of the phases on the rotor present, after a transient state, some perturbations around sinusoidal curves forming a balanced three-phased system. Likewise the torque and the speed show perturbations around constant values. Perturbations are mainly due to harmonics of the coefficients of induction between stator and rotor. The dotted lines on figure 9 and figure 10 represent the results of simulation obtained with the fundamental harmonic of these components.

During steady state operation the value of the slip can be deduced either from results on figure 9 or from results on

figure 10. In both cases the slip calculated is about 8 per cent. Relation (7), which is necessary to establish the expression of V_r (9) and the torque (14) in the Park's model, is therefore verified. From the value of the slip and from the values of the parameters (16), the value of torque obtained from Park's model is $2.6 \cdot 10^{-12}$ N.m which is roughly the value obtained by the general simulation procedure when fundamental harmonic of the coefficient of induction is considered.

The model defined in paragraph III seems to give a relatively good representation of the electromechanical behaviour of E.I.M.. This model is more adapted to the design of the machine and its supply than a more complicated model based with the electromagnetic waves theory used until now.

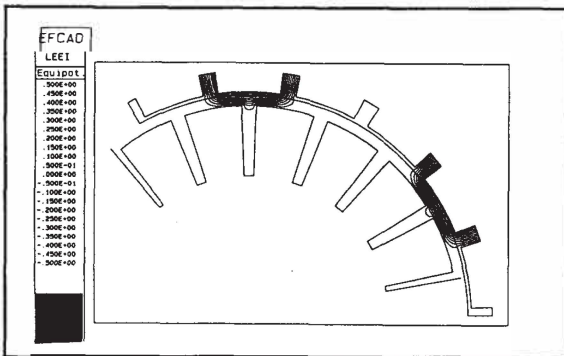


Fig. 7 Structure of the machine in simulation

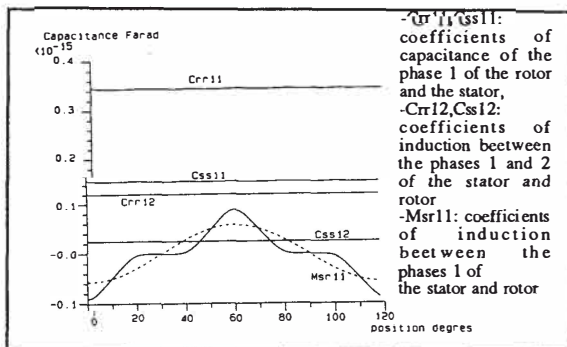


Fig. 8 Mutual and self capacitances stator and rotor

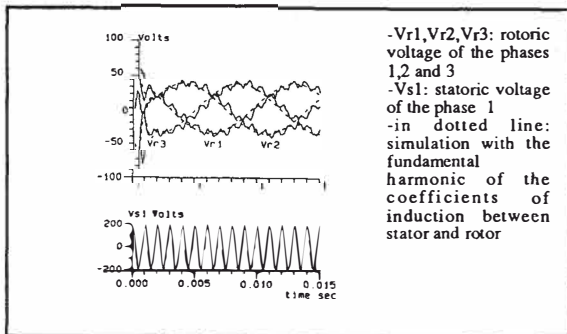


Fig. 9 Rotor and statoric voltages

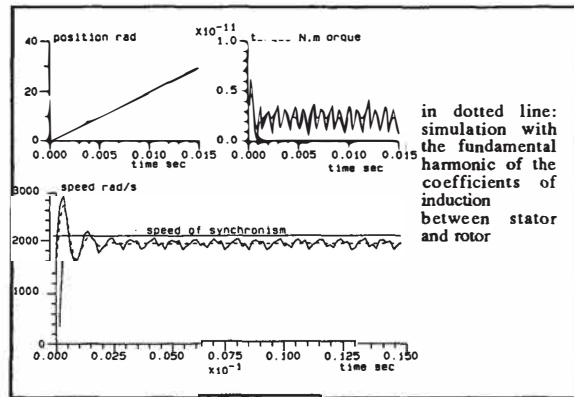


Fig. 10 Torque, speed and position

VI. CONCLUSION

At first some considerations of duality allows us to synthesize an electrostatic induction motor and we have also developed an electromechanical model of this actuators which is more simple than those developed in the previous works on this type of machine. Then we have elaborated a general procedure for the simulation of the dynamic working of these actuators. The results obtained allows us to validate our model which can be useful for the design of these machines and their supply. Results obtained must be confirmed experimentally.

REFERENCES

- [1] -W.S.N. Trimmer-, "Microrobots and micromechanical systems." Sensors and Actuators 19 (1989) pp. 267-287.
- [2] -J.H. Lang, M.F. Schletch- "Electric micromotors: Electromechanical Characteristics." Tech. Dig. I.E.E.E Microrobots and Teleoperators Workshop (Hyannis, MA, nov 1987)
- [3] -M. Mehregany, K.J. Gabriel , W.S.N. Trimmer - "Integrated Fabrication of Polysilicon Mechanism." I.E.E.E. Transaction on Electron. Devices. vol 35 n° 6 june 1988.
- [4] -P.T. Krein, J.M. Crowley- "Harmonic effects in electrostatic induction motors " Electric Machines And Power Systems, 10: pp. 479-497 1985
- [5] -S.F. Bart, J.H. Lang- "An analysis of Electrostatic induction micromotors " Sensors and Actuators 20(1989), pp.97-103
- [6] - E. Sarraute - "Etude et modélisation de microactionneurs intégrés sur silicium." Thèse de docteur INPT - Toulouse - 26 Octobre 1993.
- [7] - E. Sarraute, Y. Lefevre, B. Trannoy, M. Lajoie-Mazenc - "Méthodes de calcul du champ électrostatique appliquées à la simulation de micromoteurs à capacités variables" Colloque sur les Méthodes Industrielles de la Conception Industrielle ESIM Marseille France - 18 Juin 1993.
- [8] -E. Sarraute, Y. Lefevre ,B. Trannoy, H. Camon, M. Lajoie-Mazenc- "Micromoteurs électromagnétiques et électrostatiques. Comparaisons des performances." Capteurs intelligents et microactionneurs intégrés . Cépadués Editions ,pp. 105-108.
- [9] -N. Sadowski- " modélisation des machines électriques à partir de la résolution des équations du champ en tenant compte du mouvement et du circuit d'alimentation (logiciel EFCAD) " Thèse de docteur INP Toulouse France 1993.